Status of $t\bar{t}$ cross section predictions

Sven-Olaf Moch
Sven-Olaf.Moch@desy.de

DESY, Zeuthen

TOP 2010 – 3rd International Workshop on Top Quark Physics, Brugge, May 31, 2010
Perturbative QCD at colliders

- Hard hadron-hadron scattering
  - constituent partons from each incoming hadron interact at short distance (large momentum transfer $Q^2$)

- QCD factorization at scale $\mu$
  - separate sensitivity to dynamics from different scales

\[
\sigma_{pp \rightarrow t\bar{t}X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}X} \left( \alpha_s(\mu^2), Q^2, \mu^2 \right)
\]

- subprocess cross section $\hat{\sigma}_{ij \rightarrow t\bar{t}X}$ for parton types $i, j$
Hard scattering cross section

- Standard approach to uncertainties in theoretical predictions
  - variation of factorization scale $\mu$: \[
  d \ln \mu^2 \sigma_{pp \rightarrow X} = \mathcal{O}(\alpha_s^{l+1})
  \]

\[
\sigma_{pp \rightarrow t\bar{t}X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow t\bar{t}}(\alpha_s(\mu^2), Q^2, \mu^2)
\]

- Parton cross section $\hat{\sigma}_{ij \rightarrow t\bar{t}}$ calculable pertubatively in powers of $\alpha_s$
  - constituent partons from incoming protons interact at short distances of order $\mathcal{O}(1/Q)$

- Parton luminosity $f_i \otimes f_j$
  - proton: very complicated multi-particle bound state
  - colliders: wide-band beams of quarks and gluons
Parton luminosity at LHC

- LHC will run at 7 TeV until end of 2011 years, up to $1 \text{ fb}^{-1}$
- $t\bar{t}$ cross section is $\sim 1/4$ rate with respect to run at 14 TeV
  - larger PDF uncertainties
- Parton kinematics restricted to larger effective $\langle x \rangle = M/\sqrt{S}$
  - 100 GeV physics: small-$x$, sea partons
  - TeV scales: large-$x$
- Limited discovery reach
  - less phase space available for heavy mass states
  - use 7 TeV for accurate SM benchmarking
Top quark production

- Leading order Feynman diagrams
  \[ q + \bar{q} \rightarrow Q + \bar{Q} \]
  \[ g + g \rightarrow Q + \bar{Q} \]

- NLO in QCD
  Nason, Dawson, Ellis ‘88; Beenakker, Smith, van Neerven ‘89; Mangano, Nason, Ridolfi ‘92; Bernreuther, Brandenburg, Si, Uwer ‘04; Mitov, Czakon ‘08; . . .

- accurate to \(O(15\%)\) at LHC
Top quark production

- Leading order Feynman diagrams
  \[ q + \bar{q} \rightarrow Q + \bar{Q} \]
  \[ g + g \rightarrow Q + \bar{Q} \]

- NLO in QCD Nason, Dawson, Ellis ‘88; Beenakker, Smith, van Neerven ‘89; Mangano, Nason, Ridolfi ‘92; Bernreuther, Brandenburg, Si, Uwer ‘04; Mitov, Czakon ‘08; . . .

  - accurate to \( O(15\%) \) at LHC

Challenge

- Improve theory predictions and reduce theoretical uncertainty
  - hard scattering cross section \( \hat{\sigma}_{ij \rightarrow t\bar{t}} \)
  - parton luminosity \( f_i \otimes f_j \)
Recent theory activities

- General structure of massive QCD amplitudes
  - relate massive to massless amplitudes in limit $m \to 0$ Mitov, S.M. ‘06
  - two-loop virtual corrections to $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  - small-mass limit $m^2 \ll s, t, u$ Czakon, Mitov, S.M. ‘07
  - complete IR singularities Ferroglia, Neubert, Pecjak, Yang ‘09

- Exact virtual amplitudes
  - one-loop squared terms (NLO × NLO)
    Anastasiou, Mert Aybat ‘08; Kniehl, Merebashvili, Körner, Rogal ‘08
  - two-loop virtual corrections for $q\bar{q} \to t\bar{t}$
    (analytic, $n_f$-terms) Bonciani, Ferroglia, Gehrmann, Maitre, Studerus ‘08;
    (analytic, two-loop planar) Bonciani, Ferroglia, Gehrmann, Studerus ‘09;
    (numerical result) Czakon ‘08

- Complete NLO corrections to $t\bar{t}$ in association with jets
  - $t\bar{t} + 1$ jet at NLO Dittmaier, Uwer, Weinzierl ‘07-‘08; Melnikov, Schulze ‘10
  - $t\bar{t} + 2$ jets at NLO Bevilaqua, Czakon, Papadopoulos, Worek ‘10

Sven-Olaf Moch
Status of $t\bar{t}$ cross section predictions – p.6
Recent theory activities (cont’d)

- Threshold resummation
  - updates of cross section predictions based on resummation
    S.M., Uwer ‘08; Cacciari, Frixione, Mangano, Nason, Ridolfi ‘08; Kidonakis, Vogt ‘08; Beneke, Czakon, Falgari, Mitov, Schwinn ‘09; Ahrens, Ferroglia, Neubert, Pecjak, Yang ‘10

- Coulomb corrections
  Hagiwara, Sumino, Yokoya ‘08; Kiyo, Kühn, S.M., Steinhauser, Uwer ‘08

- Definition of mass parameter
  Hoang, Jain, Scimemi, Stewart ‘08

- Parton luminosity
  → talk by Guffanti
  - precision PDFs for LHC (NNLO global analyses)
    Martin, Stirling, Thorne, Watt ‘08; Alekhin, Blümlein, Klein, S.M. ‘09; Jimenez-Delgado, Reya ‘09
  - correlation of cross section at NLO with gluon PDFs
    Nadolsky, Lai, Cao, Huston, Pumplin, Stump, Tung, Yuan ‘08
  - benchmarking of PDFs under way
NLO

- Cross section for heavy-quark hadro-production (scaling functions $f_{ij}$)

\[
\hat{\sigma}_{ij \rightarrow t\bar{t}} = \frac{\alpha_s^2}{m_t^2} \left\{ f_{ij}^{(0)}(\rho) + (4\pi\alpha_s) f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f) \right\}
\]

- Perturbative expansion at NLO
**NLO**

- Cross section for heavy-quark hadro-production (scaling functions $f_{ij}$)

\[ \hat{\sigma}_{ij \to t\bar{t}} = \frac{\alpha_s^2}{m_t^2} \left\{ \hat{f}_{ij}^{(0)}(\rho) + (4\pi\alpha_s) \hat{f}_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f) \right\} \]

- Perturbative expansion at NLO

**Strategy beyond NLO**

- Use universal features of soft/collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft $\theta_{qg} = 0$ collinear

\[ \alpha_s \int d^4k \frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_qE_g(1 - \cos \theta_{qg})} \]

\[ \alpha_s \int dE_g d\sin \theta_{qg} \frac{1}{2E_qE_g(1 - \cos \theta_{qg})} \]

\[ \rightarrow \alpha_s \ln^2(\ldots) \]
Beyond NLO: all-order resummation

Cross section for heavy-quark hadro-production (scaling functions $f_{ij}$)

$$\hat{\sigma}_{ij \rightarrow t\bar{t}} = \frac{\alpha_s^2}{m_t^2} \left\{ f_{ij}^{(0)}(\rho) + (4\pi\alpha_s) f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f) \right\}$$

$$\qquad + f_{ij}^{\text{resummed}}(\alpha_s, N, \mu_f/m_t, \mu_r/\mu_f) + \mathcal{O}(N^{-1} \ln^n N)$$

All order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n \ln^{2n}(N)$

- resummation in Mellin space (renormalization group equation)
- long history Kidonakis, Sterman ‘97; Bonciani, Catani, Mangano, Nason ‘98; Kidonakis, Laenen, S.M., Vogt ‘01; ...

Upshot:

- $f_{ij}^{\text{resummed}} \simeq \exp\left(\alpha_s \ln^2 N\right) + \mathcal{O}(N^{-1} \ln^n N)$
Beyond NLO: NNLO\textsuperscript{approx}

Cross section for heavy-quark hadro-production (scaling functions $f_{ij}$)

$$
\hat{\sigma}_{ij \rightarrow t\bar{t}} = \frac{\alpha_s^2}{m_t^2} \{ f_{ij}^{(0)}(\rho) + (4\pi\alpha_s) f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f) + (4\pi\alpha_s)^2 f_{ij}^{(2)}(\rho, \mu_f/m_t, \mu_r/\mu_f) + O(\alpha_s^3) \}
$$

General structure at NNLO

- dependence on factorization and renormalization scale
  $$ L_M = \ln(\mu_f^2/m_t^2) \text{ and } L_R = \ln(\mu_r^2/\mu_f^2) $$

$$
\begin{align*}
  f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/m_t) &= f_{ij}^{(10)} + L_M f_{ij}^{(11)} + 2\beta_0 L_R f_{ij}^{(0)}, \\
  f_{ij}^{(2)}(\rho, \mu_f/m_t, \mu_r/m_t) &= f_{ij}^{(20)} + L_M f_{ij}^{(21)} + L_M^2 f_{ij}^{(22)} + 3\beta_0 L_R f_{ij}^{(10)} + 3\beta_0 L_R L_M f_{ij}^{(11)} + 2\beta_1 L_R f_{ij}^{(0)} + 3\beta_0^2 L_R^2 f_{ij}^{(0)}
\end{align*}
$$

- only unknown: $f_{ij}^{(20)}$ (but knowledge of threshold logarithms)

- all other function known through renormalization group equations
Two-loop results

- NNLO cross section for heavy-quark hadro-production near threshold
  S.M, Uwer ‘08; Beneke, Czakon, Falgari, Mitov, Schwinn ‘09

- e.g. $gg$-fusion for $n_f = 5$ light flavors at $\mu = m_t$

\[
f_{gg}^{(10)} = \frac{f_{gg}^{(0)}}{(16\pi^2)} \left\{ 96 \ln^2 \beta - 9.5165 \ln \beta + 35.322 + 5.1698 \frac{1}{\beta} \right\}
\]

\[
f_{gg}^{(20)} = \frac{f_{gg}^{(0)}}{(16\pi^2)^2} \left\{ 4608 \ln^4 \beta - 1894.9 \ln^3 \beta + \left(-912.35 + 496.30 \frac{1}{\beta}\right) \ln^2 \beta + \left(2456.7 + 321.14 \frac{1}{\beta}\right) \ln \beta + 68.547 \frac{1}{\beta^2} - 8.6226 \frac{1}{\beta} + C_{gg}^{(2)} \right\}
\]

Who’s who

- Sudakov logarithms in $\ln \beta$ (generated from resummed cross sections)

- $1/\beta$-terms from Coulomb corrections
  resummation to all orders
  in non-relativistic QCD

- unkown constant $C_{gg}^{(2)}$
Total cross section at Tevatron

\[ \sigma_{pp \to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij \to t\bar{t}} \]

\[ \text{Luminosity } L_{ij} \left[ \text{1/GeV}^2 \right] \]
\[ \sqrt{s} = 1.96 \text{ TeV} \]
\[ \text{CTEQ 6.5} \]
\[ \mu_f = \mu_r = 171 \text{ GeV} \]

\[ \Delta L_{q\bar{q}} \left[ \% \right] \]
\[ \Delta L_{gg} \left[ \% \right] \]
\[ \Delta L_{qg} \left[ \% \right] \]

\[ \text{NLO QCD Partonic cross section [pb]} \]
\[ \mu_f = \mu_r = 171 \text{ GeV} \]

\[ \text{NLO QCD Hadronic cross section [pb]} \]
\[ \mu_f = \mu_r = m_t = 171 \text{ GeV} \]
Scale dependence

- Theoretical uncertainty from variation of scales $\mu_R, \mu_F$
- Plot with PDF set MSTW 2008 (but largely independent on PDFs)
- Mass $m_t = 173$ GeV (from $m_t = 173.1 \pm 1.3$ GeV Tevatron winter '09)
- Stable predictions in range $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - $-3% \leq \Delta \sigma \leq +1\%$ at LHC
  - $-5% \leq \Delta \sigma \leq +3\%$ at Tevatron

\[ \mu_r/\mu_f \quad \text{[pb]} \]

- LHC
- MSTW 2008 NNLO
- $m = 173$ GeV

\[ \mu_r/\mu_f \quad \text{[pb]} \]

- Tevatron
- MSTW 2008 NNLO
- $m = 173$ GeV
The total cross section

- Theory prediction at NNLO$_{\text{approx}}$ accuracy
  - pole mass $m_t = 173$ GeV
  - theoretical uncertainty from variation of $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - NNLO PDF set and $\alpha_s$, e.g. MSTW 2008

\[
\begin{align*}
\sigma_{\text{LHC}} &= 157.0 \text{ pb} \quad \pm_{-6.5}^{+2.1} \text{ pb (scale)} \quad \pm_{-4.4}^{+4.4} \text{ pb (MSTW2008)} \\
\sigma_{\text{TeV}} &= 6.93 \text{ pb} \quad \pm_{-0.32}^{+0.15} \text{ pb (scale)} \quad \pm_{-0.14}^{+0.14} \text{ pb (MSTW2008)}
\end{align*}
\]
The total cross section

- Theory prediction at NNLO_{approx} accuracy
  - pole mass $m_t = 173$ GeV
  - theoretical uncertainty from variation of $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - NNLO PDF set and $\alpha_s$, e.g. MSTW 2008

\[
\sigma_{LHC} = 157.0 \text{ pb} \quad \pm 2.1 \quad \pm 4.4 \text{ pb (scale)} \quad \pm 4.4 \text{ pb (MSTW2008)}
\]

\[
\sigma_{TeV} = 6.93 \text{ pb} \quad \pm 0.15 \quad \pm 0.14 \text{ pb (scale)} \quad \pm 0.14 \text{ pb (MSTW2008)}
\]

Quality control

- Check of systematics: variation of $C_{gg}^{(2)}$
  - recall $f_{gg}^{(20)} \simeq f_{gg}^{(0)} \{ 4608 \ln^4 \beta + \cdots + C_{gg}^{(2)} \}$
  - Estimate systematic uncertainty of total cross section $\Delta \sigma \sim \mathcal{O}(2\%)$
    - $\Delta \sigma \sim \mathcal{O}(3 - 4) \text{ pb}$ at LHC 7 TeV
    - $\mathcal{O}(10 - 15) \text{ pb}$ at 14 TeV
    - $\Delta \sigma \sim \mathcal{O}(0.15 - 0.2) \text{ pb}$ at Tevatron
**tt+ jet production (I)**

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO corrections to $t\bar{t}+$jet production are part of NNLO corrections for inclusive $t\bar{t}$ production

Check of systematics: hard radiation
- at scale $\mu_R = \mu_F = m_t$ corrections are almost zero
- threshold resummation captures dominant contributions

\[ \sigma [\text{pb}] \]

\[ pp \rightarrow t\bar{t}+\text{jet}+X \]
\[ \sqrt{s} = 14 \text{ TeV} \]
\[ p_T, \text{jet} > 20 \text{GeV} \]
### $t\bar{t} + \text{jet}$ production (II)

<table>
<thead>
<tr>
<th>$p_T,\text{jet,cut}[\text{GeV}]$</th>
<th>$\sigma_{t\bar{t}\text{jet}}[\text{pb}]$</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$1.583(2)^{+0.96}_{-0.55}$</td>
<td>$1.791(1)^{+0.16}_{-0.31}$</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>$0.984(1)^{+0.60}_{-0.34}$</td>
<td>$1.1194(8)^{+0.11}_{-0.20}$</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>$0.6632(8)^{+0.41}_{-0.23}$</td>
<td>$0.7504(5)^{+0.072}_{-0.14}$</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>$0.4670(6)^{+0.29}_{-0.17}$</td>
<td>$0.5244(4)^{+0.049}_{-0.096}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_T,\text{jet,cut}[\text{GeV}]$</th>
<th>$\sigma_{t\bar{t}\text{jet}}[\text{pb}]$</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$710.8(8)^{+358}_{-221}$</td>
<td>$692(3)3^{+40}_{-62}$</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>$326.6(4)^{+168}_{-103}$</td>
<td>$376.2(6)^{+17}_{-48}$</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>$146.7(2)^{+77}_{-47}$</td>
<td>$175.0(2)^{+10}_{-24}$</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>$46.67(6)^{+26}_{-15}$</td>
<td>$52.81(8)^{+0.8}_{-6.7}$</td>
<td></td>
</tr>
</tbody>
</table>

Cross section $\sigma_{t\bar{t}\text{jet}}$ for different values of $p_T,\text{jet,cut}$ for $\mu = \mu_R = \mu_F = \{m_t/2, m_t, 2m_t\}$ with PDF sets CTEQ6L1, CTEQ6M.

Dittmaier, Uwer, Weinzierl '07-'08

Tevatron

LHC 14
Concordance approach
Cacciari, Czakon, Mangano, Mitov, S.M., Nason, Uwer [to appear]

- Philosophy
  - genuine NLO approach improved with threshold logarithms only
  - technically: truncation of \( f_{ij}^{(2)} = f_{ij}^{(20)} + L_M f_{ij}^{(21)} + L_M^2 f_{ij}^{(22)} + \ldots \)
to logarithmic accuracy in \( \ln \beta \)

- Conservative estimate of theoretical uncertainty
  - scale variation of \( \mu_R, \mu_F \in [m_t/2, 2m_t] \) with \( 1/2 \leq \mu_R/\mu_F \leq 2 \)
  - e.g. pole mass \( m_t = 173 \text{ GeV} \) and NNLO PDF set MSTW08

\[
\sigma_{LHC} = 155.4 \text{ pb}^{+1.3}_{-6.3} \text{ pb (scale)}^{+4.4}_{-4.4} \text{ pb (MSTW2008)}
\]

\[
\sigma_{TeV} = 6.92 \text{ pb}^{+0.01}_{-0.41} \text{ pb (scale)}^{+0.14}_{-0.14} \text{ pb (MSTW2008)}
\]
C++ package
Aliev, Lacker, Langenfeld, S.M. Uwer [to appear]

Cross section evaluation done entirely in Hathor class

```cpp
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;
double mt = 171., muf=171., mur=171.;
double val, err, chi2a, up, down;
Lhapdf pdf("MSTW2008nnlo68cl");
Hathor XS(pdf)
```

PDF choice
HATHOR

C++ package
Aliev, Lacker, Langenfeld, S.M. Uwer [to appear]

Cross section evaluation done entirely in Hathor class

```
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;
double mt = 171., muf=171., mur=171.;
double val,err,chi2a,up,down;
Lhapdf pdf("MSTW2008nnlo68cl");  ← PDF choice
Hathor XS(pdf)
XS.setPrecision(Hathor::MEDIUM);
XS.getXsection(mt,mur,muf);  → σ = 164.3^{+4.6}_{-9.2} pb (scale unc.)
XS.getResult(0,val,err,chi2a);
```
C++ package
Aliiev, Lacker, Langenfeld, S.M. Uwer [to appear]

Cross section evaluation done entirely in Hathor class

```c++
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;
double mt = 171., muf=171., mur=171.;
double val, err, chi2a, up, down;
Lhapdf pdf("MSTW2008nnlo68cl"); ← PDF choice
Hathor XS(pdf)
XS.setPrecision(Hathor::MEDIUM);
XS.getXsection(mt, mur, muf); → σ = 164.3^{+4.6}_{-9.2} pb (scale unc.)
XS.getResult(0, val, err, chi2a);
XS.setScheme(scheme | Hathor::PDF_SCAN); ← with PDF error scan
XS.setPrecision(Hathor::LOW);
XS.getXsection(mt, mur, muf);
XS.getPdfErr(up, down);
} → σ = 164.3^{+4.6}_{-9.2} pb (sc.) ^{+4.4}_{-4.4} pb (PDF unc.)
```

Sven-Olaf Moch
Dependence on parton distributions

- Comparison of NLO and NNLO
- Theoretical uncertainty from scale variation: $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - uncertainties: scale (left) and scale + PDF (right)
- Sizeable difference between the ABKM/MSTW sets at NNLO well outside the PDF uncertainty
  - due to value of $\alpha_s$ and shape of gluon PDFs at $\langle x \rangle = 2m_t/\sqrt{s}$
  - can only be settled with first LHC data

![Graph showing differences in ttbar cross section predictions at NLO and NNLO between ABKM_5 and MSTW08 sets with and without PDF uncertainties.](chart.png)
Dependence on parton distributions

- Comparison of NLO and NNLO\textsubscript{approx}
- Theoretical uncertainty from scale variation: $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - uncertainties: scale (left) and scale + PDF (right)
- Sizeable difference between the ABKM/MSTW sets at NNLO well outside the PDF uncertainty
  - due to value of $\alpha_s$ and shape of gluon PDFs at $\langle x \rangle = 2m_t/\sqrt{s}$
  - can only be settled with first LHC data
Resummation of Coulomb corrections

- Invariant mass distribution $d\sigma/dM_{t\bar{t}}$
- At LHC $gg \rightarrow t\bar{t} \left(1S_0^1\right)$ dominates; driven by large gluon luminosity
- At Tevatron with small bound state effects; $q\bar{q}$-channel large with only color-octet configurations only

![Graphs showing the invariant mass distribution at LHC and Tevatron](image_url)
Electroweak corrections

Electroweak corrections (ratio of $\sigma_{EW}/\sigma_{LO}$) Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth ‘94; Bernreuther, Fücker ‘05; Kühn, Uwer, Scharf ‘06

Effect depends on Higgs mass (choices $m_H = 120\,\text{GeV}, m_H = 200\,\text{GeV}, m_H = 1000\,\text{GeV}$)

LHC 14

Tevatron

Tevatron: vanishing contribution for light Higgs

LHC: $O(2\%)$ with respect to $\sigma_{LO}$

negative contribution to total cross section $\Delta\sigma_{EW} \sim O(10 - 15)\,\text{pb}$
Mass dependence of cross section

Pole mass scheme

Based on (unphysical) concept of top-quark being a free parton

\[ \phi - m_t - \Sigma(p, m_t) \bigg|_{p^2 = m_t^2} \]

- heavy-quark self-energy \( \Sigma(p, m_t) \) receives contributions from regions of all loop momenta – also from momenta of \( \mathcal{O}(\Lambda_{QCD}) \)
Mass dependence of cross section

Pole mass scheme

- Based on (unphysical) concept of top-quark being a free parton

\[ p - m_t - \Sigma(p, m_t) \]

- heavy-quark self-energy \( \Sigma(p, m_t) \) receives contributions from regions of all loop momenta – also from momenta of \( \mathcal{O}(\Lambda_{\text{QCD}}) \)

- Pole mass measurements are strongly order-dependent

  - e.g. threshold scan of cross section in \( e^+ e^- \) collision
  - Beneke, Signer, Smirnov ‘99;
  - Hoang, Teubner ‘99;
  - Melnikov, Yelkhovsky ‘98;
  - Penin, Pivovarov ‘99;
  - Yakovlev ‘99

  - LO (dotted), NLO (dashed), NNLO (solid)
Tevatron analyses

- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- Determination of $m_t$ from total cross section (slope $d\sigma/dm_t$)
  - e.g. DZero '09: NLO $m_t = 165.5^{+6.1}_{-5.9}$; NNLO $m_t = 169.1^{+5.9}_{-5.2}$, ...

![Graph showing the cross section as a function of top mass with measurements and predictions at various orders of accuracy.](image-url)
The running top-quark mass

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
- short distance mass probes at scale of hard scattering
- conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m_t = m(\mu_R) \left( 1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)} \right)$
- Scale dependence greatly reduced

![Graph showing $\sigma$ vs $\mu_f/\mu_t$ for LHC and Tevatron](image)

Sven-Olaf Moch
The running top-quark mass

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
- short distance mass probes at scale of hard scattering
- conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory:

$$m_t = m(\mu_R) \left( 1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)} \right)$$

- Pole mass scheme for comparison

![Graph of $\sigma$ vs $\mu_r/\mu_f$ for LHC and Tevatron]

LHC
MSTW 2008 NNLO
$m = 173$ GeV

Tevatron
MSTW 2008 NNLO
$m = 173$ GeV
Mass dependence in $\overline{MS}$ mass scheme

- Total top-quark cross section as function of $\overline{m}$
- Theoretical uncertainty (band) due to variation of $\mu_R \in [\overline{m}/2, 2\overline{m}]$ for fixed set $\mu_F \in \overline{m}/2, \overline{m}, 2\overline{m}$

LHC

- MSTW 2008 NNLO

Tevatron

- MSTW 2008 NNLO

First direct determination of $m(m)$ Langenfeld, S.M., Uwer ‘09

$\overline{MS}$ mass $m(m) = 160.0^{+3.3}_{-3.2}$ GeV

Conversion to pole mass $m_t = 168.9^{+3.5}_{-3.4}$ GeV
Implications for indirect Higgs searches

- Electroweak precision data constrains $M_H$

- pole mass $m_t = 168.9^{+3.5}_{-3.4}$ GeV
  (lighter top-quark masses disfavor SM Higgs sector)
Summary

- Top quark theory
  - much recent progress for Tevatron and LHC phenomenology
  - improved understanding of theory and application of new concepts

- Total cross section
  - $\text{NNLO}_{\text{approx}}$ prediction with exact scale dependence $\mu_R \neq \mu_F$
    - $(\ln(\mu_R/m), \ln(\mu_F/m))$-terms
  - cross check on systematics with NLO correction to $t\bar{t} + \text{jet}$
  - electroweak corrections
  - bound state effects for $t\bar{t}$-system

- $\overline{MS}$ mass definition
  - running top-quark mass of $m(\mu = m) = 160.0^{+3.3}_{-3.2}$ GeV
  - greatly reduced scale dependence
  - much improved convergence of perturbation theory
Extra slides
Other recent developments

- Total cross section from invariant mass distribution $d\sigma/dM_{t\bar{t}}$
  Ahrens, Ferroglia, Neubert, Pecjak, Yang ‘10
  - resummation of threshold logarithms in $\ln(1 - M_{t\bar{t}}^2/s)/(1 - M_{t\bar{t}}^2/s)$

- Cross section numbers significantly lower
  - $\sigma_{\text{LHC}} = 146 \text{ pb} \pm 7 \text{ pb (scale)} \pm 8 \text{ pb (MSTW2008)}$
  - $\sigma_{\text{TeV}} = 6.48 \text{ pb} \pm 0.17 \text{ pb (scale)} \pm 0.32 \text{ pb (MSTW2008)}$

- Open issues:
  - impact of scale choices $\mu_s, \mu_h, \mu_f$
  - compatibility at high energy (BFKL asymptotics at small-$x$)

- Kidonakis, Laenen, S.M., Vogt ‘01